Oxford Cambridge and RSA

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Practice Paper – Set 1

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \frac{u}{v}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0}\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\sum(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\sum x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\dfrac{\sum fx^2}{\sum f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, Mean of X is np, Variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Section A: Pure Mathematics

Answer **all** the questions

1 **In this question you must show detailed reasoning.**

Find the gradient of the curve $y = 3\cos 2x$ at the point where $x = \frac{1}{8}\pi$. **[4]**

2 **(i)** Express $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[3]**

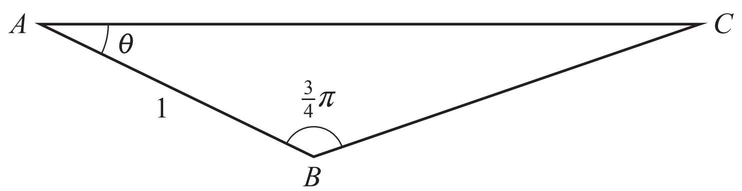
The temperature θ °C of a building at time t hours after midday is modelled using the equation

$$\theta = 20 + 4\cos(15t)° + 3\sin(15t)°, \text{ for } 0 \leqslant t < 24.$$

(ii) Find the minimum temperature of the building as given by this model. **[1]**

(iii) Find also the time of day when this minimum temperature occurs. **[3]**

3



The diagram shows triangle ABC, in which angle $A = \theta$ radians, angle $B = \frac{3}{4}\pi$ radians and $AB = 1$ unit.

(i) Use the sine rule to show that $AC = \dfrac{1}{\cos\theta - \sin\theta}$. **[3]**

(ii) Given that θ is a small angle, use the result in part **(i)** to show that

$$AC \approx 1 + p\theta + q\theta^2,$$

where p and q are constants to be determined. **[4]**

4 **In this question you must show detailed reasoning.**

It is given that the geometric series

$$1 + \frac{5}{3x-4} + \left(\frac{5}{3x-4}\right)^2 + \left(\frac{5}{3x-4}\right)^3 + \dots$$

is convergent.

(i) Find the set of possible values of x, giving your answer in set notation. **[5]**

(ii) Given that the sum to infinity of the series is $\frac{2}{3}$, find the value of x. **[3]**

5 **(i)** By sketching the graphs of $y = \frac{5}{x^2}$ and $y = |2-4x|$ on a single diagram, show that the equation

$$\frac{5}{x^2} = |2-4x| \qquad \text{(A)}$$

has exactly two real roots. **[3]**

(ii) Show that the positive root α of equation (A) satisfies the equation $\mathrm{f}(x) = 0$, where

$$\mathrm{f}(x) = 4x^3 - 2x^2 - 5.$$ **[1]**

(iii) Hence show that a Newton-Raphson iterative formula for finding α can be written in the form

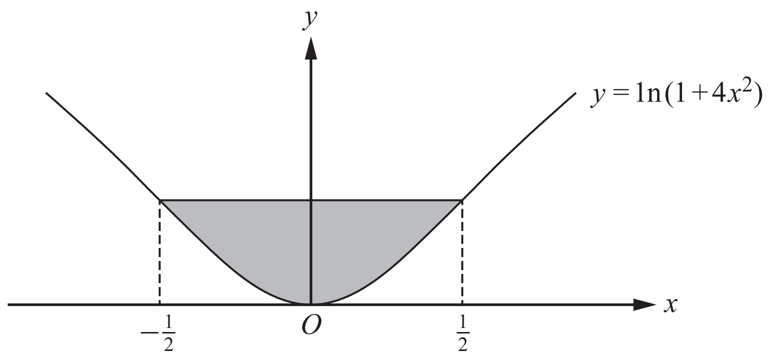
$$x_{n+1} = \frac{8x_n^3 - 2x_n^2 + 5}{12x_n^2 - 4x_n}.$$ **[3]**

(iv) Use this iterative formula, with initial value $x_1 = 1$, to find the value of α correct to 3 decimal places. Show the result of each iteration. **[3]**

A student claims that the iterative formula from part **(iii)** can be used to find the negative root of equation (A) provided that a suitable initial value is chosen.

(v) Explain why the student's claim is incorrect. **[1]**

6 **(i)** Show that the two non-stationary points of inflection on the curve $y = \ln(1+4x^2)$ are at $x = \pm\frac{1}{2}$. **[6]**



The diagram shows the curve $y = \ln(1+4x^2)$. The shaded region is bounded by the curve and a line parallel to the x-axis which meets the curve where $x = \frac{1}{2}$ and $x = -\frac{1}{2}$.

(ii) Show that the area of the shaded region is given by

$$\int_0^{\ln 2} \sqrt{e^y - 1}\, dy.$$ **[3]**

(iii) Show that the substitution $e^y = \sec^2\theta$ transforms the integral in part **(ii)** to $\int_0^{\frac{1}{4}\pi} 2\tan^2\theta \, d\theta$. **[2]**

(iv) Hence find the exact area of the shaded region. **[3]**

Section B: Mechanics

Answer **all** the questions

7 A monorail train travels along a straight horizontal track from station A to station B. The train accelerates uniformly from rest at A to a maximum speed of $30\,\mathrm{m\,s^{-1}}$ then travels at this speed for 90 seconds before slowing down uniformly to come to rest at B. The acceleration of the train is $a\,\mathrm{m\,s^{-2}}$, the deceleration is $2a\,\mathrm{m\,s^{-2}}$ and the time for the whole journey is 3 minutes.

(i) Sketch the velocity-time graph for the journey. **[2]**

(ii) Calculate the distance between the two stations. **[2]**

(iii) Calculate the value of a. **[3]**

8 A particle P of mass 3 kg moves under the action of a force $\begin{pmatrix} 9 \\ -3 \end{pmatrix}$ N. Initially P has velocity $\begin{pmatrix} 1 \\ -2 \end{pmatrix}\mathrm{m\,s^{-1}}$ and is at the point with position vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ m. At time t seconds later, P has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$.

(i) Express $\mathbf{v}$ in terms of t. **[2]**

(ii) Find the value of t when the speed of P reaches $5\,\mathrm{m\,s^{-1}}$. **[3]**

(iii) Find the position vector of P when $t = 2$. **[2]**

9 A boy kicks a ball from a point O on horizontal ground. The ball first hits the ground at a distance of 60 m from O and the time of flight is 4 seconds. This motion of the ball is modelled as that of a particle moving freely under gravity.

(i) Find the horizontal and vertical components of the initial velocity of the ball. **[3]**

The ball just clears a vertical post, of height h m, at a horizontal distance of 15 m from O.

(ii) Show that $h = 14.7$. **[2]**

(iii) Find the speed of the ball as it passes over the post. **[4]**

Measurements show that the speed of the ball as it passes over the post is in fact not equal to the value found in part **(iii)**.

(iv) State a deficiency of the model that might account for this. **[1]**

(v) Explain whether an improved model would require a larger or smaller initial speed for the ball. **[1]**

10 A particle P of weight W lies on the surface of a rough plane which is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{4}{3}$. The coefficient of friction between the particle and the plane is $\frac{1}{2}$. A horizontal force of magnitude H is applied to P. This force acts in the vertical plane through a line of greatest slope. It is given that H is the greatest value for which P remains in equilibrium.

(i) Indicate on a diagram the forces acting on P. **[1]**

(ii) Show that $H = \frac{11}{2}W$. **[5]**

The horizontal force acting on P is now removed.

(iii) Find the acceleration of P in terms of g. **[4]**

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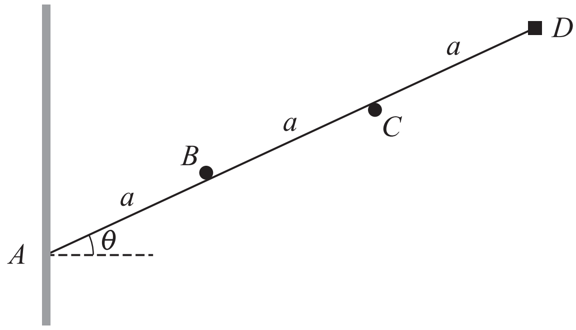

A thin light rod AD has length $3a$. The end A is in contact with a smooth vertical wall which is perpendicular to the vertical plane containing the rod. The rod carries a load of weight W at the end D. The rod is held in equilibrium by two fixed smooth pegs B and C, where $AB = BC = CD = a$. The rod passes under peg B and over peg C, and makes an angle θ with the horizontal (see diagram).

(i) **(a)** Show that the normal contact force at C may be expressed as $W\left(\frac{3\cos^2\theta - 1}{\cos\theta}\right)$. **[5]**

(b) Find the normal contact force at B in terms of W and θ. **[1]**

(ii) Hence show that the value of θ is at most 35.3°, correct to 3 significant figures. **[2]**

(iii) Show that it is not possible for the magnitude of the reaction at A to equal the magnitude of the reaction at C. **[6]**

END OF QUESTION PAPER