Exercise 13.1S

1 Copy and complete these calculations to show the different ways that 24 can be written as a product of its factors.

a $24 = \square \times 2$ **b** $24 = 3 \times \square$

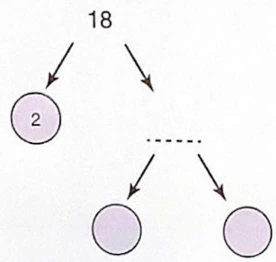
c $24 = 2 \times 3 \times \square$ **d** $24 = 4 \times \square$

2 Each of these numbers has just two prime factors, which are not repeated. Write each number as the product of its prime factors.

a 77 **b** 51 **c** 65

d 91 **e** 119 **f** 221

3 Copy and complete the factor tree to find the prime factor decomposition of 18.



4 Work out the values of these expressions.

a $3^2 \times 5^2$ **b** $2^3 \times 5$

c $3^2 \times 7^2$ **d** $2^2 \times 3 \times 5^2$

5 Write each number as the product of powers of its prime factors.

a 36 **b** 120 **c** 34

d 48 **e** 27 **f** 105

g 99 **h** 37 **i** 91

6 Write the prime factor decomposition for each of these numbers.

a 1052 **b** 2560 **c** 825

d 715 **e** 1001 **f** 219

g 289 **h** 2840 **i** 2695

j 1729 **k** 3366 **l** 9724

m 11 830 **n** 2852 **o** 10 179

7 The diagram below shows how Laura found the LCM of 8 and 6.

Multiples of 8 = 8, 16, 24, 32, 40, 48 ...
Multiples of 6 = 6, 12, 18, 24, 30, 36 ...

Use Laura's method to find the LCM of 12 and 9.

8 Using the method from question **7**, find the LCM of these pairs of numbers.

a 4 and 5 **b** 12 and 18

c 5 and 30 **d** 12 and 30

e 14 and 35 **f** 8 and 20

9 Find the HCF of each pair of numbers, by drawing a Venn diagram or otherwise.

a 35 and 20 **b** 48 and 16

c 21 and 24 **d** 25 and 80

e 28 and 42 **f** 45 and 60

10 Find the LCM of each pair of numbers, by drawing a Venn diagram or otherwise.

a 24 and 16 **b** 32 and 100

c 22 and 33 **d** 104 and 32

e 56 and 35 **f** 105 and 144

11 Find the LCM and HCF of these numbers.

a 180 and 420 **b** 77 and 735

c 240 and 336 **d** 1024 and 18

e 762 and 826 **f** 1024 and 1296

12 Find the HCF and LCM of these numbers.

a 30, 42, 54 **b** 90, 350, 462

c 462, 510, 1105 **d** 44, 57, 363

13 **a** Show that 33 105 is divisible by 15.

b Show that 262 262 is divisible by 1001.

14 A number is **abundant** if its factors, excepting the number itself, sum to a total greater than the number. If this total is less than the number it is **deficient** and if the total equals the number it is **perfect**.

For example 12 has factors 1, 2, 3, 4, 6 (and 12) $1 + 2 + 3 + 4 + 6 = 16$ and $16 > 12$ so 12 is abundant. Complete the table below by classifying the following numbers, the first one has been done for you.

12, 72, 40, 86, 30, 50, 64, 6, 27, 28

Abundant	12
Perfect	
Deficient	

13.1 Factors and multiples

RECAP

- The **highest common factor** (HCF) of two numbers is the largest number that is a factor of them both.
- The **least common multiple** (LCM) of two numbers is the smallest number that they both divide into.
- You can find the HCF and LCM of two numbers by writing their **prime factors** in a Venn diagram.
  - The HCF is the product of the numbers in the intersection.
  - The LCM is the product of all the numbers in the diagram.

HOW TO

To solve problems involving factors or multiples.

1. RTQ – decide how to use your knowledge of factors and multiples.
2. Be systematic – use listing, factor trees and/or Venn diagrams to help you to find multiples and factors.
3. ATQ – make sure that you explain your answers fully.

EXAMPLE

Explain why the square of a prime number has exactly three factors.

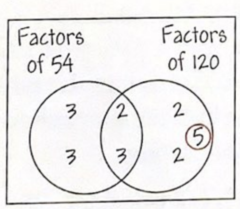
1. A prime number, p, has two factors, 1 and p (itself).
2. List the factors of the square of a prime number.
   $p^2 = 1 \times p^2$ and $p^2 = p \times p$
3. The only factors are: 1, the prime number, p, and the number itself, p^2.

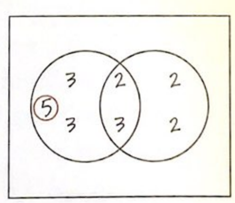
EXAMPLE

The highest common factor of two numbers is 6.
The lowest common multiple is 1080.
Billie says that the two numbers must be 54 and 120.
Show that there are other possibilities.

1. Find another pair of numbers with the same HCF and LCM.
2. Write the prime factors of 54 and 120 in a Venn diagram.

$54 = 2 \times 3^3$ $\qquad$ $120 = 2^3 \times 3 \times 5$





You can move the factors around to find another pair of numbers with the same HCF and LCM, but the numbers in the intersection have to stay the same.

3. Find another pair of numbers with the same HCF and LCM.
   A possible pair of numbers is
   $2 \times 3^3 \times 5 = 270$ and $2^3 \times 3 = 24$
   Other possible pairs are $2 \times 3 = 6$ and $2^3 \times 3^3 \times 5 = 1080$
   or $3^3 \times 2^3 = 216$ and $2 \times 3 \times 5 = 30$.

Exercise 13.1A

1 Explain why the cube of a prime number has exactly four factors.

2 Recall the definition of a deficient number from exercise **13.1S**. Use algebra to show that the square of a prime must be deficient.

3 The highest common factor of two numbers is 30. The lowest common multiple is 900. Omar says that the two numbers must be 150 and 180. Show that there is another possibility.

4 Two numbers have HCF = 15 and LCM = 90. One of the numbers is 30. What is the other number?

5 Look at this statement.

> The product of any two numbers is equal to the product of their HCF and their LCM.

a Test the statement for three pairs of numbers. Do you think it is true?

b Use a Venn diagram to justify your answer to part **a**.

6 a Amos says 'all odd numbers are prime numbers'. Give two examples that show he is wrong.

b Aya says 'all prime numbers are odd numbers'. Give one example to show that she is wrong.

c Arik says 'take one off any multiple of 6 and you always get a prime number'.

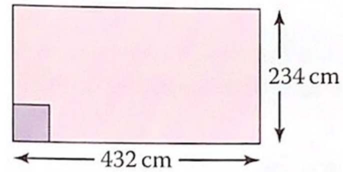
i Give 3 examples where this is true.

ii Give 1 example where it is false.

7 Emily checks her phone every 10 minutes to see if her friend Charlotte is available to chat. Charlotte checks hers every 8 minutes. They both decide to check at 9:00 am. When is the first time after this that they are both online?

8 Two needles move around a dial. The faster needle moves around in 24 seconds and the slower needle in 30 seconds. If the two needles start together at the top of the dial, how many seconds does it take before they are next together at the top?

9 A wall measures 234 cm by 432 cm. What is the largest size of square tile that can be used to cover the wall, without needing to cut any of the tiles?



10 The number 18 can be written as $2 \times 3 \times 3$. You can say that 18 has three prime factors.

a Find three numbers with exactly three prime factors.

b Find five numbers with exactly four prime factors.

c Find four numbers between 100 and 300 with exactly five prime factors.

d Find a two-digit number with exactly six prime factors.

11 A cuboid has a volume of $1815\,\text{cm}^3$. Each side of the cuboid is a whole number of centimetres and each side is longer than 1 cm. Find all the possible dimensions of the cuboid.

12 Find the HCF and LCM of $x^2 - 1$ and $2x^2 - x - 1$.

13 The ratio $x:y$ is $3:7$.
Find the following in terms of x.

a The HCF of $9y + x$ and $3y + 4x$.

b The LCM of $9y + x$ and $3y + 4x$.

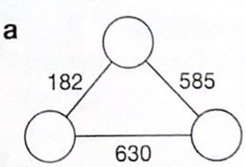
14 How many factors do these numbers have?

a 2^3 b $2^2 \times 3^3$ c $5^2 \times 7^3$

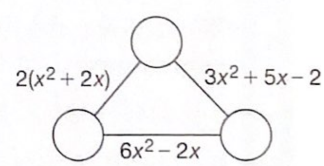
d 625 e 45 f 484

15 The product of each pair of circled expressions is given on the edge between them. Copy the diagrams and fill in the missing expressions.

a



b



3 **a** 2^7 **b** 3^3 **c** 5^7 **d** 6^2
e 7^3 **f** 4^4

4 **a** 1 **b** 1 **c** 5 **d** 0.5

13.1S

1 **a** 12 **b** 8 **c** 4 **d** 6

2 **a** $77 = 7 \times 11$ **b** $51 = 3 \times 17$
c $65 = 5 \times 13$ **d** $91 = 7 \times 13$
e $119 = 7 \times 17$ **f** $221 = 13 \times 17$

3 9, 3, 3; $18 = 2 \times 3 \times 3$

4 **a** 225 **b** 40 **c** 441 **d** 300

5 **a** $36 = 2^2 \times 3^2$ **b** $120 = 2^3 \times 3 \times 5$
c $34 = 2 \times 17$ **d** $48 = 2^4 \times 3$
e $27 = 3^3$ **f** $105 = 3 \times 5 \times 7$
g $99 = 3^2 \times 11$ **h** $37 = 37$
i $91 = 7 \times 13$

6 **a** $2^2 \times 263$ **b** $2^9 \times 5$
c $3 \times 5^2 \times 11$ **d** $5 \times 11 \times 13$
e $7 \times 11 \times 13$ **f** 3×73
g 17^2 **h** $2^3 \times 5 \times 71$
i $5 \times 7^2 \times 11$ **j** $7 \times 13 \times 19$
k $2 \times 3^2 \times 11 \times 17$ **l** $2^2 \times 11 \times 13 \times 17$
m $2 \times 5 \times 7 \times 13^2$ **n** $2^2 \times 23 \times 31$
o $3^3 \times 13 \times 29$

7 36

8 **a** 20 **b** 36 **c** 30 **d** 60
e 70 **f** 40

9 **a** 5 **b** 16 **c** 3 **d** 5
e 14 **f** 15

10 **a** 48 **b** 800 **c** 66 **d** 416
e 280 **f** 5040

11 **a** 1260, 60 **b** 8085, 7 **c** 1680, 48 **d** 9216, 2
e 314 706, 2 **f** 82 944, 16

12 **a** HCF = 6, LCM = 1890 **b** HCF = 2, LCM = 34 650
c HCF = 1, LCM = 510 510 **d** HCF = 1, LCM = 27 588

13 **a** The digits sum to a multiple of 3 so it is divisible by 3 and the last digit is 5 so it is divisible by 5.
b The three-digit sequence 262 is repeated in this six-digit number. 262 262 = 262(1000 + 1)

14 Abundant: 72, 40, 30 Perfect: 6, 28 Deficient: 86, 50, 64, 27

13.1A

1 Let p be prime. The only factorisations possible are $p^3 = 1 \times p^3 = p \times p^2$ so the only factors of p^3 are p^3, p^2, p and 1.

2 The factors of the square of a prime p^2 are 1, p and p^2. Sum of proper factors = $1 + p$.
$1 + p < p^2$ if $0 < p^2 - p - 1$ for any prime p

b

Let A,
sectio
$P = A$
HCF =
as requ

6 **a** 9, 15, ..
b 2 (the
c **i** 6 −
(the
ii 36 −

7 Both onlin

8 120 second

9 18 cm

10 **a** e.g. 8, 1
b e.g. 16,
c e.g. 108
d e.g. 64

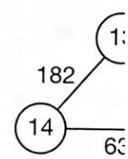
11 15 × 11 ×

12 HCF = x −
LCM = (x

13 **a** $11x$

14 **a** 4
e 6

15 **a**



13.2S

1 **a** 4.5 (1 dp
b **i** 6.3 (1

2 **a** **i** 2.7 (1

3 **a** ±36.67 (
c ±84.22 (

4 **a** 23

5 **a** 6^5
e 3^{12}

6 **a** 7^2
e 1

7 **a** 3^4
e 3^{18}

8 **a** 8^5

number. 262 262 = 262(1000 + 1)

14 Abundant: 72, 40, 30   Perfect: 6, 28   Deficient: 86, 50, 64, 27

13.1A

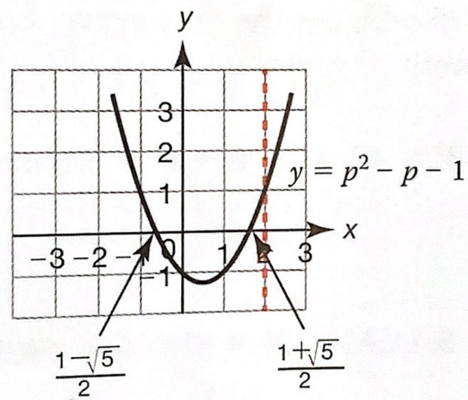
1 Let p be prime. The only factorisations possible are $p^3 = 1 \times p^3 = p \times p^2$ so the only factors of p^3 are p^3, p^2, p and 1.

2 The factors of the square of a prime p^2 are 1, p and p^2. Sum of proper factors $= 1 + p$.

$1 + p < p^2$ if $0 < p^2 - p - 1$ for any prime p.

From the graph, $0 \geqslant p^2 - p - 1$ for $\frac{1 - \sqrt{5}}{2} \leqslant p \leqslant \frac{1 + \sqrt{5}}{2}$ only,

and there are no prime numbers in this region so we have the result.

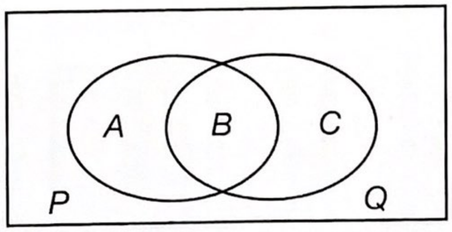


3 Another possibility is 450 and 60.

4 45

5 **a** Any pair of numbers will support the statement.

b



Let A, B and C be the products of the numbers in each section.

$P = A \times B, Q = B \times C, P \times Q = A \times B^2 \times C$

HCF $= B$, LCM $= A \times B \times C$, HCF $\times$ LCM $= A \times B^2 \times C$ as required.

6 **a** 9, 15, ... (lots more)

b 2 (the only one that isn't odd)

c **i** $6 - 1 = 5$, $12 - 1 = 11$, $18 - 1 = 17$ (there are lots more)

ii $36 - 1 = 35$ (there are more)

7 Both online at 9:40.

8 120 seconds

9 18 cm

10 **a** e.g. 8, 12, 105

b e.g. 16, 24, 36, 54, 81

c e.g. 108, 112, 120, 162

d e.g. 64

11 $15 \times 11 \times 11$, $33 \times 5 \times 11$, $55 \times 3 \times 11$

12 HCF $= x - 1$

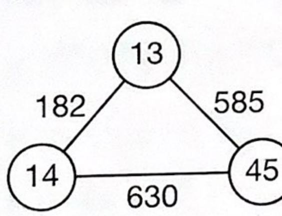
LCM $= (x + 1)(x - 1)(2x + 1) = 2x^3 + x^2 - 2x - 1$

13 **a** $11x$ **b** $22x$

14 **a** 4 **b** 12 **c** 12 **d** 5

e 6 **f** 9

15 **a**



b